Application of Singular Spectrum Analysis to solar irradiance variability

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Abstract. Studies of solar variability improve our knowledge of the internal structure and dynamical processes taking place within the Sun that lead to solar irradiance changes. Because of the astrophysical and climatic significance of irradiance variability, considerable effort has been devoted to model and understand its physical origin. However, earlier analyses did not provide adequate information on the number of variables needed to explain and predict the observed changes and they were unable to characterize the noise in the data. The main purpose of this paper is to study the noise characteristics of total solar irradiance and to identify the major components of the s]]ort-term irradiance changes using a new technique: Singular Spectrum Analysis.

1. Introduction

Various space experiments within the last one and half decades demonstrated that total solar irradiance varies over a wide range of periodicities on both long and short time scales (Fröhlich, 1994). The long-term irradiance variations over the solar cycle are attributed to the changing emission of bright magnetic elements, including faculae and the magnetic network (Foukal & Lean, 1 988). The short-term variations from days to months are directly related to the evolution of active regions (Fröhlich & Pap, 1989). The most striking events in the short-term changes of total irradiance are the sunspot-related temporary dips wit h amplitudes up to 0.3% (Willson *et al.*, 1981).

Although considerable information exists on irradiance variability, the underlying physical mechanisms are not yet understood. 'l'he current empirical models of total irradiance, developed with linear regression analyses (e.g., Foukal & Lean, 1988; Pap et. *al.*, 1994), cannot explain all aspects of the observed changes. These regression models underestimate the observed irradiance during the maximum of solar cycles 21 and 22, causing considerable uncertainties in es-intimating and predicting the long-term changes which are important for climate studies. We note that the regression models can only give overall information on the observed irradiance changes and they neglect that solar total irradiance and its surrogates are "pseudo-periodic" signals with variable spectral properties (e.g. Fröhlich & Pap, 1989; Bouwer, 1992). It has been shown that the variations in total irradiance can be studied more successfully in the frequency domain, using multivariate spectral analysis. Results of multivariate spectral analysis (Fröhlich & Pap, 1989) show that considerable variation remains unexplained in total irradiance after removing the effect of sunspots and bright magnetic features at periods of 9, 13.5, and 51 days, and the amplitude of this residual variability depends on the phase of the solar cycle. However, it is not clear whether the unexplained variations are related to the noise in the data or they represent additional events contributing to irradiance changes.

Before designing an empirical model, the first question to ask is how many variables are required to reconstruct the observed changes up to the measurement accuracy. This issue implies the necessity of determining the degrees of freedom of the variability of the examined system, or in other words, to determine the statistical dimension of the dataset. It is also essential to choose the most appropriate basis functions) rather than to fix them, before understanding the dynamics of the physical system.

In this paper we present the first results of Singular Spectrum Analysis applied to study the variability of total solar irradiance observed by the ACRIM 1 radiometer on the Solar Maximum Mission (SMM) satellite in 1980 and its relation to the evolution of active regions. The present work has been stimulated by the theory of nonlinear dynamical systems and chaos. In the last decades considerable new information has been accumulated on the subject and there are now techniques designed to deal with actual observed data as opposed to data generated by computer simulations (Ott *et al.*, 1994).

2. A new approach in statistical modeling: Singular Spectrum Analysis

An advanced statistical method, Singular Spectrum Analysis (SSA) has been developed and applied to study and understand nonlinear and chaotic dynamical systems (Vautard *et al.*, 1992). It complements the theory of such systems with a statistical tool to extract as much information as possible from short and noisy data. SSA has numerous advantages compared to formerly used techniques. SSA performs as a data adaptive filter, instead of using fixed basis functions, therefore it is highly capable of distinguishing between the dominant oscillations of the system and clarifying the noise characteristics of the data. This method allows us to model the former part of variations in great detail and to isolate the parts of irradiance changes of the greatest interest.

SSA is based on Principal Component Analysis in the time domain. The examined dataset is augmented into a number of shifted time series up to a fixed value M, called window length. The basis of SSA is to calculate the M x M lag-covariance matrix between the shifted time series. The so-called Singular Spectrum is created from the eigenvalues of this matrix which are arranged in a monotonically decreasing order. The Singular Spectrum levels off after a certain order by forming a noise floor. The number of eigenvalues above the noise

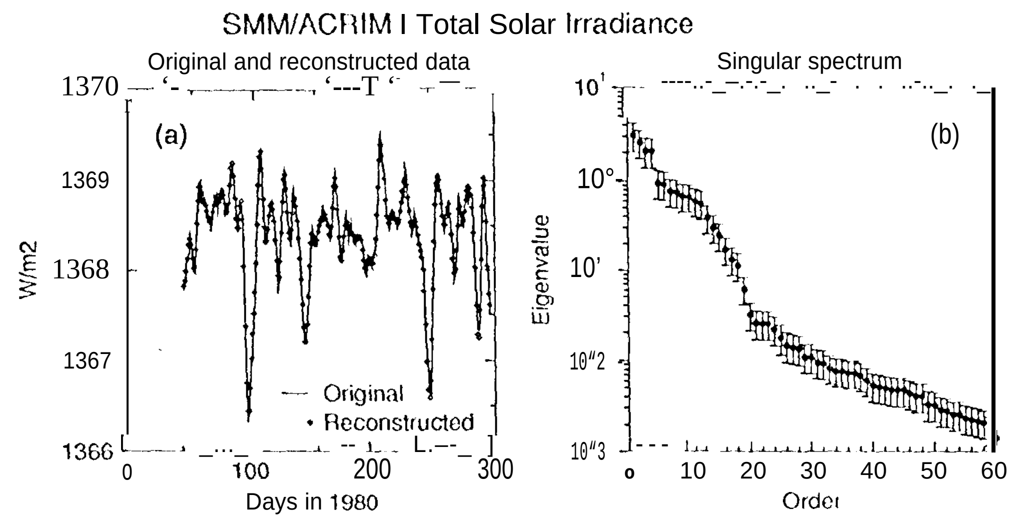

Figure 1. The time series of the ACRIM I total solar irradiance is plotted in Fig. 1a during the normal operation of the SMM satellite in 1980. The corresponding Singular Spectrum is presented in Fig. 1b.

floor represents the statistical dimension of the investigated time series and they are associated with deterministic components in the signal. The highest order eigenvalue is related to the most significant variability in the examined time series and in many cases it represents the trend in the data. The eigenvectors of the lag-covariance matrix give the so-called temporal empirical orthogonal functions (T-EOFs).

The original dataset above the noise level and/or only parts of interest can be reconstructed as a projection to certain T-EOFs. These "Reconstructed Components" are usually associated with particular oscillations belonging to certain eigenvalues. We note that the number of the resolved components, representing various oscillations in the data, are determined by the M window length. To reconstruct strange attractors, larger M is better, but. to avoid the statistical errors, the window length should be less than one third of the length of the investigated time series. Further details on SSA are provided by Vautard *et al.* (1992).

3. Reconstructed **short-term** changes of total **irradiance and sunspots**

in this section the sutq)ot-related short-term changes in total solar irradiance are analyzed by means of SSA. For this purpose we have chosen the normal operational mode of the SMM/ACRIM I data in 1980, when the measuring precision of ACRIM I was so high that all the observed events had solar origin (Willson *et al.*, 1981). The SMM/ACRIM 1 time series and its Singular Spectrum with M = 60 window length are plotted in Figs. 1 a and b, respectively. As can be seen, as many as 20 degrees of freedom of the ACRIM 1 data are established from the eigenvalues above the noise level. The first 20 components of the ACRIM I data have been reconstructed and they are plotted on the top of the measured irradiance values (Fig. 1 a). The good agreement between the observed and reconstructed irradiance data indicates that the first 20 components represent the observed variability and the noise contributes to only a very small fraction of the observed irradiance signal in 1980.

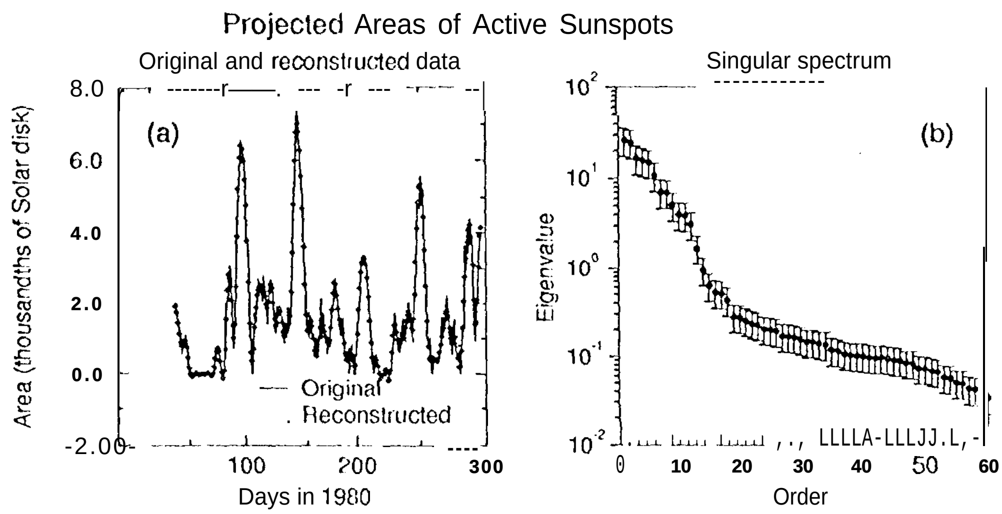

Figure 2. The projected areas of active sunspots are presented in Fig. 2a. The solid line shows the data collected from the Solnech. Dannye Catalogue. The symbol o represents the reconstructed components above the noise level. The corresponding Singular Spectrum is plotted on the right-side panel (b).

To reveal information on the effect of the evolution of active regions on total irradiance, the projected areas of "active" and "passive" sunspots have been analyzed with SSA. The active spots have been defined by Pap (1985) as fast developing complex sunspots with γ and/or δ magnetic configurations associated with newly emerging magnetic activity. The passive spots are old simple spots with α or β magnetic configurations. The time series of the projected areas of active and passive spots are plotted in Figs. 2a and 3a, respectively; the corresponding Singular Spectra are presented in Figs. 2b and 3b. Figs. 2a and 3a show that the largest dips in total irradiance are associated with the peaks in the active spot areas. In contrast, the area of passive spots is higher when peaks occur in the ACRIM I data. As can be seen from Figs. 2b and 3b, the Singular Spectra of the active and passive spots are also quite different. in the case of the active spots, the noise level is much lower than for the passive spots. This is related to the larger measuring uncertainty of the areas of passive spots, which are in general much smaller than the complex active sunspots. As many as 18 degrees of freedom of the changes in the active spot areas are established, whereas about 14 degrees of freedom are found in the case of the passive spots, indicating that a smaller number of oscillations can be identified from their areas.

The individual components of the ACRIM I total irradiance, representing

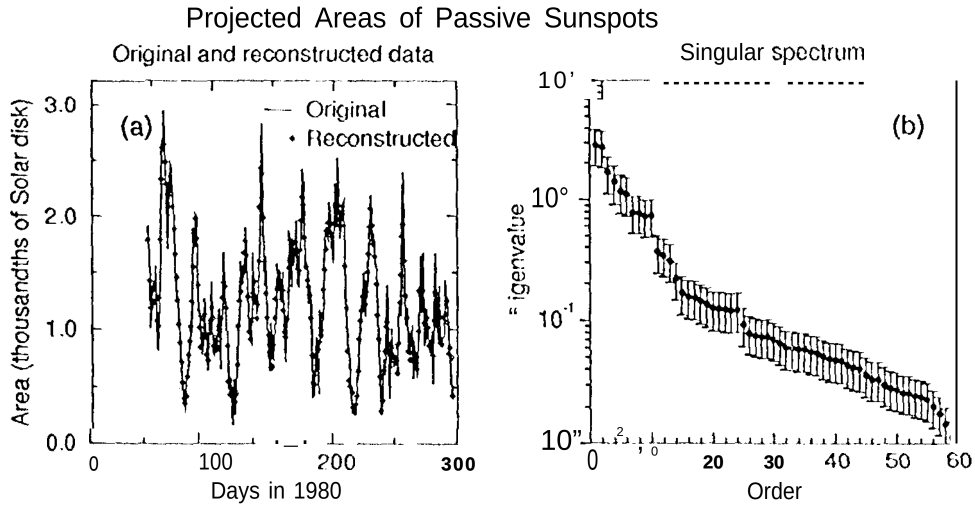

Figure 3. The same as in Fig. 2 but for the passive spots.

the most significant oscillations between periods 10 and 51 days, have been reconstructed and plotted in Fig. 4. In comparison, Figs. 5 and 6 show the same for the projected areas of active and passive spots, respectively. As can be seen from Fig. 4, the first four Reconstructed Components (hereafter RC) in the ACRIM 1 data represent the 50 and 25 days oscillations associated with sunspots. Peaks with periods of 25 and 50 days are also isolated in the first five RCs of the projected areas of active spots (Fig. 5). An oscillation with a period of approximately 16 days is clearly seen in both total irradiance (RC5-6) and in the active spot area (RC6-8), which appears to be harmonic to the 50-day oscillation. The origin of RC7-8 and RC9-10 in total irradiance is less clear. RC7-8 represents an oscillation with a period of about 11 days and every second dip in this RC corresponds to a dip in RC2-4 (25-day oscillation). This suggests that RC7-8 is a harmonic of the 25-day oscillation. RC9-10 represents, on average, the 13-day oscillation in the ACRIM 1 data, but it does not seem to be in phase with any of the separated oscillatory components. This indicates that the 13-day variability in the ACRIM total irradiance represents an independent oscillation, rather than a resonance. In the case of the active spots, RC9-11 is a mixture of a variability with 11 to 13 days and it seems to be associated with the 25-day oscillation,

As can be seen from Fig. 6, the Reconstructed Components of the passive spots are quite different from those of total irradiance and active sunspot area. The largest eigenvalue in the Singular Spectrum of the passive spots represent a 30-day variability. It is interesting to note that this oscillation almost disappeared in April and May when a number of new active regions, causing the first dips in the ACRIM 1 data, were formed on the Sun. The 30-day oscillation is restored in June with the declining new activity and it is seen in the rest of the investigated time interval in 1980. RC3 represents a trend which has a maximum in July, when mainly old active regions dominated the solar surface.

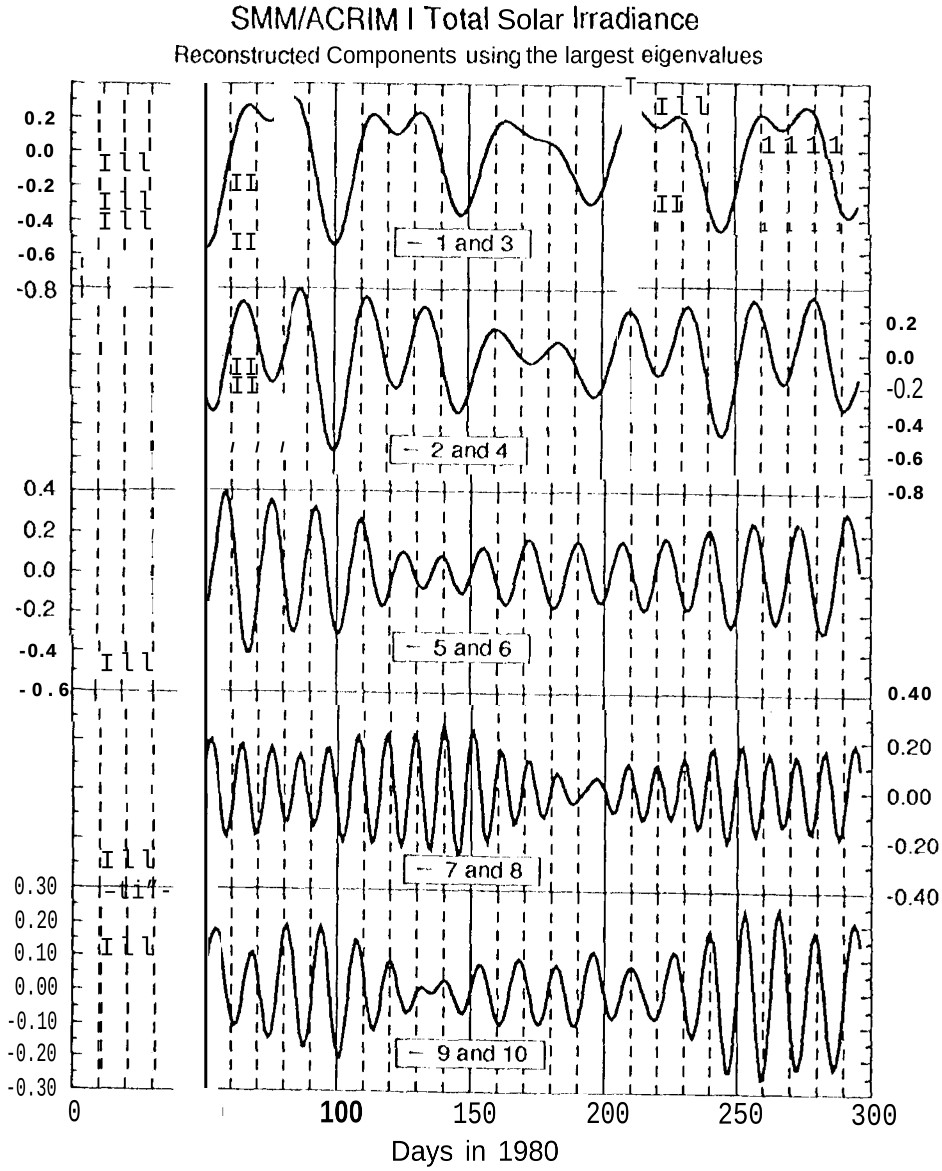

Figure 4. The Reconstructed Components of the SMM/ACRIM 1 total solar irradiance up to order 10.

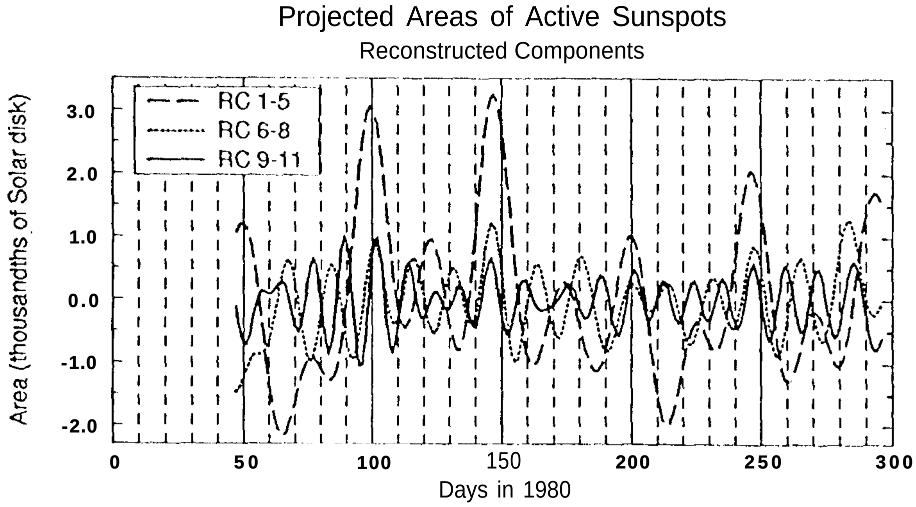

Figure 5. The Reconstructed Components of the projected areas of active sunspots.

RC4-6 in the projected areas of passive spots can be identified as a 20-day oscillation, however, its amplitude changes over time. The change of its amplitude is anticorrelated with the temporal variation of RC3. RC4-6 is clearly present by mid-May (Day 125), when its length between successive peaks decreases to about 16 days. The original 20- day oscillation re-appears in early September.

4. Conclusions

Results of Singular Spectrum Analysis demonstrate that the dominant variations of total irradiance with 11, 13, 16, 25 and 50 days periods are related to new magnetic activity represented by the active sunspot groups. There is a clear difference between the temporal behavior of various components reconstructed in the projected area of active and passive spots. While in the case of active spots, similar to total irradiance, the examined oscillations are present over the entire time interval (February to December 1980) with stable periodicity, the oscillations are more unstable in the case of passive spots. in this latter case there is a trend, indicating that the old activity was most dominant in mid-1980.

Many of the frequencies found in the SMM/ACRIM I total irradiance and projected areas of the active spots are similar to those of other solar events, such as flare activity, proton events, radio fluxes. These frequencies are close to integral multiples, suggesting that they are subharmonic of a fundamental frequency (Bai & Sturrock, 1991). A nonlinear dynamical system with a periodic forcing term can show periodic behavior at subharmonic frequencies. In the future we plan a systematic study of solar variability in terms of nonlinear resonance locking and breaking between various modes. These studies will lead to a better understanding of the solar dynamo, the governing force of solar variability and the solar-induced climate changes.

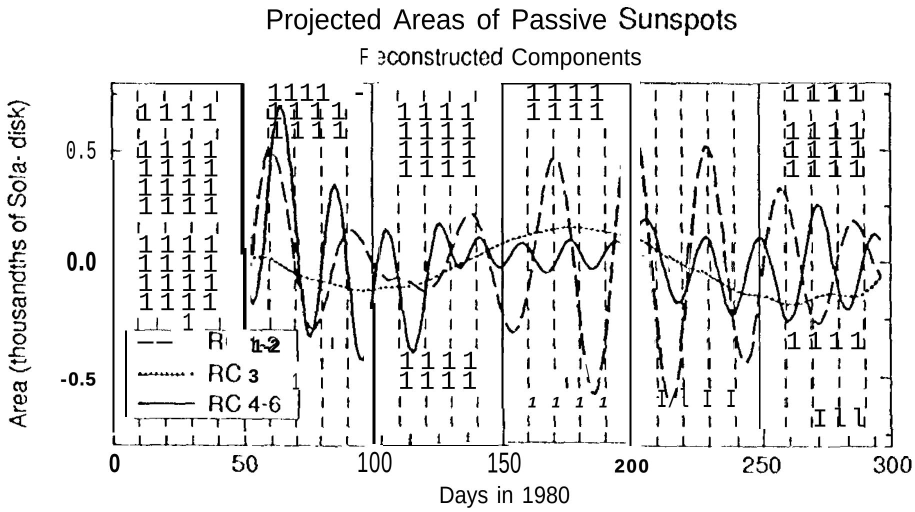

Figure 6. The Reconstructed Components of the projected areas of passive sunspots.

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